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FATIGUE LIFE STUDY OF THE KINETIC SCULPTURE BLADE

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Keywords: kinetic sculpture, fatigue design, life prediction

1. Introduction

Len Lye was born in Christchurch, New Zealand in 1901. Lye's interests in avant-garde art led him to Sydney, Samoa, London, and finally to New York where he settled until his death in 1980. Lye's interests lay mainly in kinetic sculptures and experimental film techniques. The kinetic sculptures that Lye prototyped were rarely completed to the scale at which he intended.

"My work, I think, is going to be pretty good for the 21st century. Why the 21st? Simply [I don't think] that there will be the means until then to have what I want, which is enlarged versions of my work."
Len Lye 1968

Today, many of Lye's visions are coming to fruition in New Zealand thanks to the Len Lye Foundation, the University of Canterbury and the advancements in engineering design knowledge and technology.

The University of Canterbury is currently working on two of Lye's sculptures – *Blade* and *Sun, Land and Sea*. This paper will deal exclusively with the work carried out for *Blade* in determining the largest economic size of the sculpture. The procedure used in determining the predicted life cycle of the sculpture can be applied to any flexible structure in a public environment subject to vibration such as aerials, towers or other sculpture such as Len Lye's Wind Wand.

2. Historical background and motivation

The kinetic sculpture *Blade* consists of a tall strip of metal that is excited into natural vibratory modes and interacts with an adjacent ball and wand as in Figure 1. The blade is fixed in a rigid clamp which slides in the horizontal plane on four linear bearings. The clamp is driven by a concealed DC motor drive which provides oscillating motion of the clamp. The blade material responds at specific oscillation frequencies by forming various mode shapes. The wand and ball are enticed into contact with the blade as the amplitude of vibration increases. The result is a captivating performance of sound, light and movement.

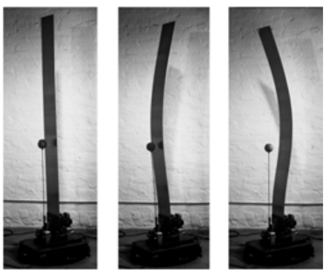

Figure 1. Len Lye's portrait of his prototype *Blade* (courtesy of the Len Lye Foundation)

The model of *Blade* that Lye built in the 1970s consists of a cold-rolled high-carbon steel strip measuring 1630 mm x 200 mm x 1,85 mm. Lye's intention was to recreate this sculpture to stand at a height of approximately 9 m [Gooch 2001], [Raine and Gooch 1998]. Blade was built and commissioned under a research contract at approximately twice (2.12) the model scale by Gooch [2001] in 1997 at the University of Canterbury. Gooch developed scaling laws that allowed the prediction of important geometry and variables such as forces and stresses in the blade material of a scaled sculpture. Gooch also determined that the constraining criteria on the allowable size of the sculpture are:

1. The bending stress in the blade material at the clamp exit, resulting in fatigue failure
2. The availability of the material for the blade with the required dimensions

Scaling of *Blade* requires static and dynamic similarity to be maintained for the artistic intentions of the work to be conserved. This results in an increased bending stress in the blade material as the size of the sculpture increases [Raine and Gooch 1998].

The material chosen by Gooch for the scaled *Blade* was Titanium 6Al-4V (Grade 5). This choice overcame the high bending stresses that occur in a sculpture that utilises a carbon steel blade due to the low elastic modulus of titanium. This was a significant design decision that allowed *Blade* to be built at a larger size whilst maintaining the artistic intentions of the work.

Upon completion of the scaled *Blade* (Figure 2a), Gooch suggested the use of an alternative clamp design to reduce bending stress in the blade material. This design is presented as a schematic in Figure 2b.

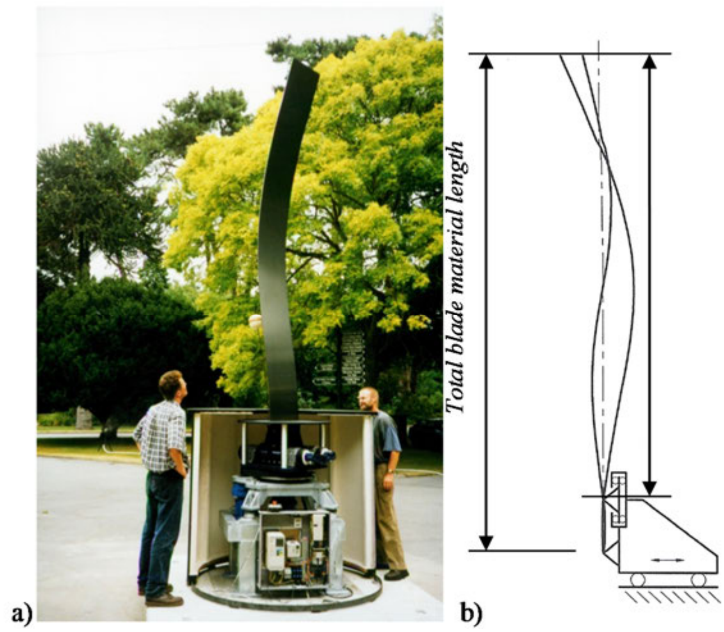

Figure 2. a) The scaled *Blade* by Gooch and b) the pivoting clamp schematic [Gooch 2001]

In this design, separated pivot points allow a larger radius of curvature to exist in the blade material at the clamp exit and, hence, a lower bending stress. This alternative clamp design, coupled with advancements in materials manufacturing technology in the past 20 years, has prompted new motivation for a larger sculpture.

3. Fatigue life prediction method

An investigation into the bending stress at the clamp exit of the pivoting clamp design is required to obtain a maximum scale for *Blade* using currently available materials.

The general approach to establishing fatigue life used in this study is presented in the logical flow diagram of Figure 3.

The bending stress in the blade material at the clamp exit has been identified as the critical location for fatigue failure [Gooch 2001]. A method of predicting bending stresses in a scaled sculpture from those measured in a test rig sculpture is required.

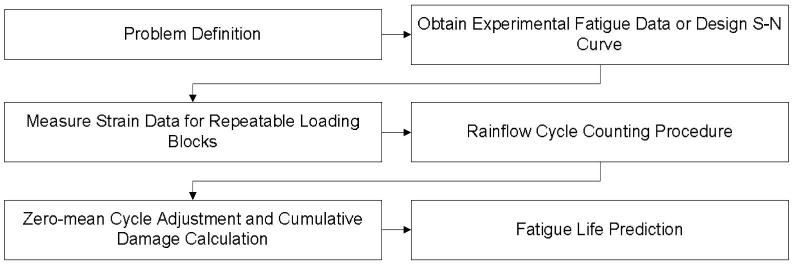

Figure 3. Flow chart showing the approach for fatigue life prediction in this study

3.1 Scaling laws

Gooch [2001] established the relationship between bending stress, material properties and size of the sculpture as follows,

$$\frac{\sigma_o}{\sigma_s} = \sqrt{\frac{\rho_o E_o l_o}{\rho_s E_s l_s}} \tag{1}$$

From equation (1), it can be shown that for similar material properties, the bending stress in the blade material increases with increasing size of the sculpture. The subscripts o and s denote the original sculpture and scaled sculpture variables respectively. σ, ρ, E, and l denote the bending stress in the blade material, the density, modulus and free length of the blade material, respectively.

3.2 Rain-flow cycle counting

Endo [Matsuishi and Endo 1968] is credited with developing the rain-flow cycle counting method. Downing and Socie [1982] present an algorithm for rain-flow cycle counting. This method of variable amplitude load history cycle counting allows extraction of all major and minor stress-strain hysteresis loops from a stress-time loading history by counting half cycles of stress during a loading history.

3.3 Smith-Watson-Topper method

Obtaining reliable fatigue data without performing lengthy fatigue tests on material samples can be difficult. Fortunately, 6Al-4V is widely used in the aerospace industry and there exists extensive data on the failure behaviour of the alloy. Dowling [2004] has also performed extensive work on equivalent zero-mean adjustments of stress cycles to allow the use of a single S-N curve in fatigue life analysis in the case of variable mean stress loading, as occurs in *Blade*. Dowling suggests the use of the Smith-Watson-Topper method for 6Al-4V in obtaining zero-mean adjusted values for stress cycles. For an S-N curve that takes the assumed form,

$$\sigma_{ar} = aN_f^b \tag{2}$$

σ_{ar} denotes the zero-mean adjusted stress cycle amplitude, a and b are the power relationship constants of the S-N curve in question, and N_f represents the number of cycles to failure at the stress amplitude in question. The SWT method states that for a non-zero-mean stress cycle, the equivalent zero-mean adjusted stress amplitude of the cycle is,

$$\sigma_{ar} = \sqrt{\sigma_{max}\sigma_a} \tag{3}$$

σ_{max} denotes the non-zero-mean adjusted stress cycle maximum value and σ_a represents the non-zero-mean adjusted stress cycle amplitude.

3.4 Titanium 6Al-4V S-N curve

Data for titanium 6Al-4V in the annealed state was obtained from the USAF High Cycle Fatigue program [United States Air Force 2001]. This set of data (Figure 4) is relevant to test conditions where the stress ratio was R=-1 (zero-mean stress cycles) and has been adjusted to fit the Smith-Watson-Topper regression by Dowling [2004].

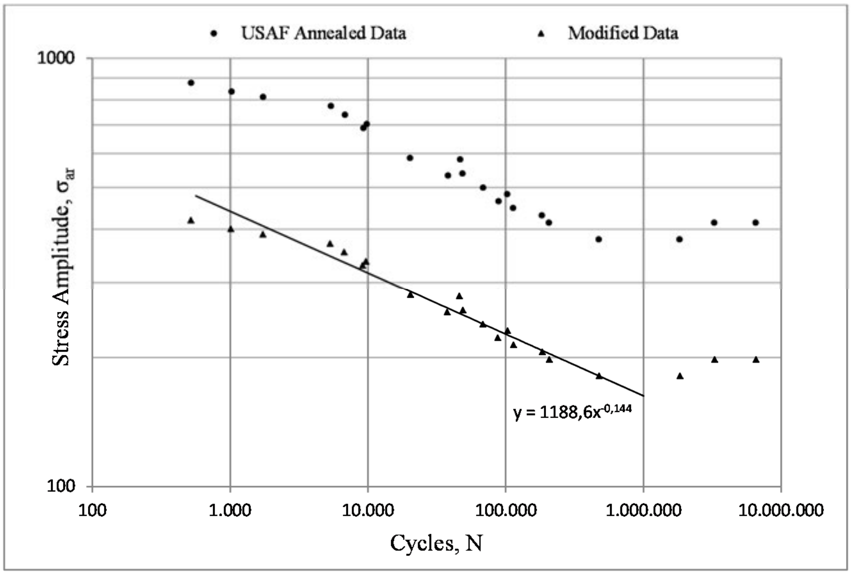

Figure 4. S-N curve for annealed 6Al-4V

A regression line has been added to the modified S-N curve (see Section 3.5 for modification factor) to determine the equivalent form of equation 2 for the curve. This is the equation used in the post-processing of measured strain data and the prediction of the fatigue life of the blade material. Cycles counted using the rain-flow counting technique were zero-mean adjusted using equation 3 for use in the S-N curve presented.

3.5 Endurance limit modification

A factor of safety on fatigue data is required due to uncertainties and irregularities that arise in the manufacture and treatment of all engineering materials. Shigley and Mischke [2001] suggest the use of endurance limit modifying factors to account for such discrepancies. Following the endurance limit modification factor method of Shigley and applying a stress concentration factor of $K_f = 1.13$ [Hosseini 2012] gives an endurance modification factor of 0.4778. As a conservative approach to this fatigue life study, the resulting endurance limit modifying factor has been applied to all S-N curve data as in Figure 4.

3.6 Modified Miner's rule for stress condition

Stress cycles that are below the endurance limit still have some effect on the life of a material [Kang et al. 2012]. In practice, the modified Miner's rule shows superior agreement over Miner's rule in experimental life testing [Kang et al. 2012], [Baek et al. 2008].

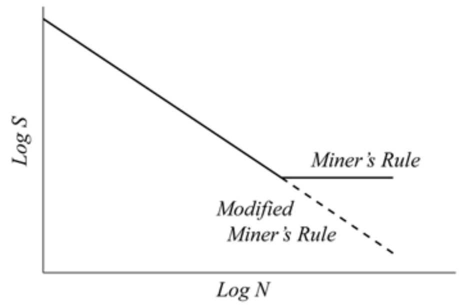

Figure 5. Modified Miner's rule is used to account for the effect of cyclic stresses with amplitudes below the endurance limit

his rule involves extrapolating the linear section of the log-log S-N curve below the endurance limit in a collinear direction as in Figure 5. Manson's method can then used to determine the damage caused to the material for all stress cycles below the endurance limit.

3.7 Manson's method for cumulative damage

Use of the Palmgren-Miner theory can lead to non-conservative life estimates [Kang et al. 2012]. Considerations for the effect of cyclic loading on the endurance limit itself and also cyclic loads below the endurance limit are required.

Once a virgin material undergoes a stress cycle above its endurance limit, the endurance limit of that material is somewhat reduced [Shigley and Mischke 2001]. This reduction is dependent on the order of the applied stress cycles. Manson's method, suggested by Shigley and Mischke [2001], is used in this study to predict the damage done, if any, to the endurance limit during each stress cycle.

The Palmgren-Miner rule assumes that the ultimate tensile strength of a material is damaged to the same extent as the endurance limit. Experiments have failed to confirm this assumption [Manson et al. 1965]. Instead, Manson's method assumes an intersection of the S-N curve at $N_0 = 10^3$ cycles and the new S-N curve after each damaging cycle pivots around this intersection. An explanation of Manson's method is presented in [Manson et al. 1965].

For the blade material specifically, if D is the damage done to the blade material during a performance, and t is the duration of a performance in seconds, then the total time, T, the blade material will last is defined as,

$$T = \frac{t}{D} \tag{4}$$

The number of performances, P_f, the blade material will last based on the loading history of the test rig is defined as,

$$P_{f,o} = \frac{T}{t} = \frac{1}{D} \tag{5}$$

The natural frequencies of the blade material also change as a function of the scale of the sculpture [Gooch 2001]. This has a direct effect on the number of stress cycles occurring in the blade material during each performance. The scaling laws developed by Gooch illustrate how natural frequencies, ω, change as the sculpture is scaled by,

$$\frac{\omega_o}{\omega_s} = \frac{d_o l_s^2}{d_s l_o^2}\sqrt{\frac{\rho_s E_o}{E_s \rho_o}} \tag{6}$$

Where, d, is the thickness of the blade material. Therefore, the number of performances that the blade material will last in the scaled sculpture is defined as,

$$P_{f,s} = P_{f,o}\frac{\omega_o}{\omega_s} \tag{7}$$

4. Predicted fatigue life of the *Blade* material

4.1 Test rig

A test rig was designed and built at the University of Canterbury at the scale of the original prototype *Blade* (Figure 6). The test rig carriage is mounted on two ground steel shafts with four bronze bushes. The carriage is shuttled by a 7.5HP electric machine. Simple harmonic motion of the carriage is achieved by ensuring the length of the connecting rod is much larger than the crank offset. The carbon steel test rig blade material measures 1934 mm x 200 mm x 1,8 mm. This size allows a consistent blade free length of 1599 mm through a range of pivot separations from 100 mm to 300 mm in 20 mm increments. A separate study has identified that the stress in the blade material at the clamp exit is

minimised at a pivot separation of 300mm. This corresponds to 19 % of the blade free length.
The lower pivot consists of a slotted shaft allowing movement of the blade in the vertical direction during blade material deflection. The upper pivot shaft clamps the blade material in place via jaws with a parabolic profile to minimise stress concentration at the clamp exit. The pivot shafts are mounted in ball bearings which allow the pivots to rotate during blade material deflection.

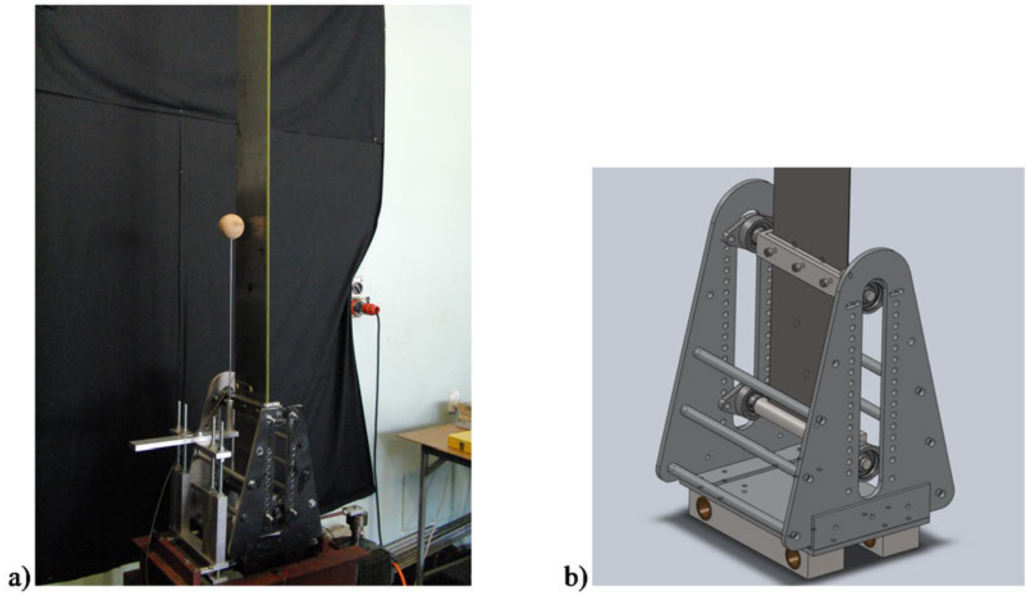

Figure 6. a) *Blade* test rig at the University of Canterbury and b) 3D CAD model of oscillating clamp

4.2 Load history measurement

Strain gages (N11-FA-2-120-11) were mounted to the surface of the carbon steel blade at the clamp exit as shown in Figure 7. The gages were mounted in a half-bridge circuit on opposing surfaces of the blade material to measure fully reversed bending strain.
The strain gages were calibrated using Elastica theory and a shunt calibration. Elastica theory [Wang 1986] describes the mechanics of long slender beams in bending. A simple cantilever test was performed and the measured strain compared to the Elastica theoretical result. The stress values were found to agree to an accuracy of within 5 %.
A shunt calibration consists of simulating a strain by passing a resistor of known resistance across a bridge arm in the strain gage circuit. The shunt calibration method in [Vishay Precision Group 2013] was used to calibrate the strain gages in the *Blade* test rig.

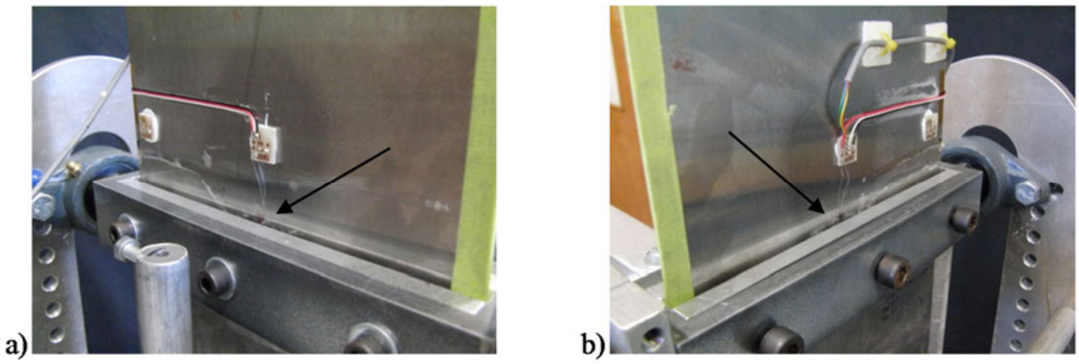

Figure 7. Strain gages mounted to the surface of the test rig blade, arrows indicate locations

LABVIEW was used to measure and process strain data in addition to controlling the test rig electric motor. The load history was measured in real time during a typical *Blade* performance.

Hooke's law via a MATLAB function was used to convert the strain-time history of the test rig performance into a stress-time history as shown in Figure 8. A MATLAB script was then used to post process the stress-time history using the theory presented in Section 3. An example of the resulting stress cycle history of a *Blade* performance is presented in Figure 8. Half cycles were counted by the rain-flow method according to the magnitude of their range. The amplitude and maximum value of each half cycle were recorded and the zero-mean equivalent stress of the half cycle was calculated using the Smith-Watson-Topper method. Manson's method was then used to determine the damage done during a typical loading block (1 performance in the case of the sculpture).

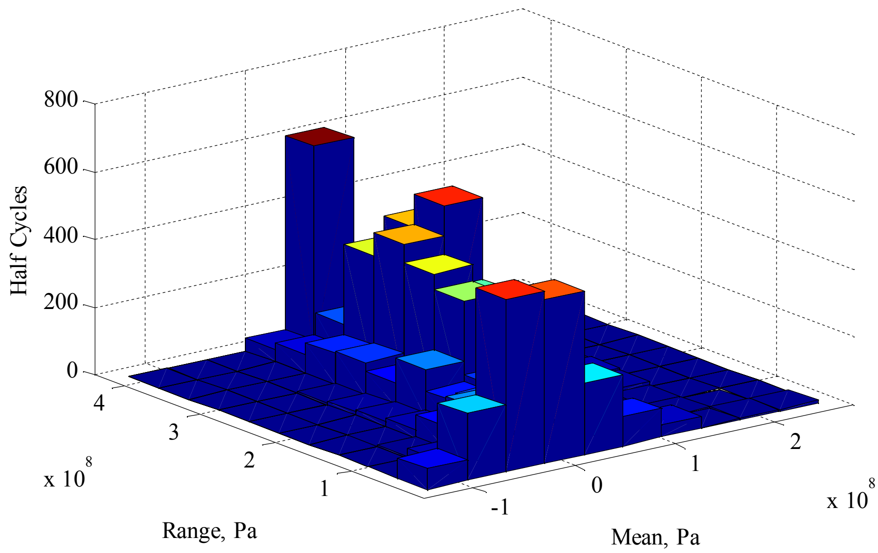

Figure 8. Stress cycle histogram for largest *Blade* free length of 8.424 m

4.3 Fatigue life prediction

Ten performances of the test rig were carried out and the load history for each performance was measured. The load history was scaled for various blade material sizes ranging from approximately 5 m to 8.5 m of free blade length. Fatigue lives were calculated for each of the ten performances according to the free blade length. The mean, standard deviation and the sample coefficient of variation (SCOV) of the ten fatigue lives were calculated at each free blade length. The method of Dowling [2007] was then used to determine one-sided tolerance limit fatigue lives for each scale ratio for 95 % confidence and 99 % reliability. These predicted fatigue lives are presented in Table 1.

Table 1. Predicted fatigue lives of the scaled sculpture at various blade material free lengths

Free Blade Length, l_s (m)	Total Blade Material Length (m)	Blade Thickness (mm)	Performances to Fatigue Failure, $P_{f,s}$	Cost Per Performance Chinese Source ($NZD)	Cost Per Performance US Source ($NZD)
8.424	10.024[1]	22	384	$163.91	N/A
8.167	9.718	21	420	$134.44	N/A
7.905	9.407	20	462	$109.07	N/A
7.64	9.091	19	509	$87.83	N/A
7.369	8.769[2]	18	567	$69.50	$118.93
7.094	8.441	17	633	$54.48	$93.23
6.813	8.107	16	712	$42.05	$71.95
6.526	7.766	15	809	$31.83	$54.47

6.232	7.416	14	925	$23.70	$40.56
5.932	7.059	13	1069	$17.25	$29.52
5.624	6.692	12	1231	$12.43	$21.27
5.307	6.315	11	1482	$8.43	$14.42
4.980	5.926	10	1789	$5.59	$9.56

1. Maximum blade material size available in China
2. Maximum blade material size available in the United States

Figure 9 shows the various predicted fatigue lives of blade material for increasing scale ratio of the sculpture. Figure 9 also shows the price per performance of each scale of blade material based solely on the price of titanium per kilogram at the time of this study.

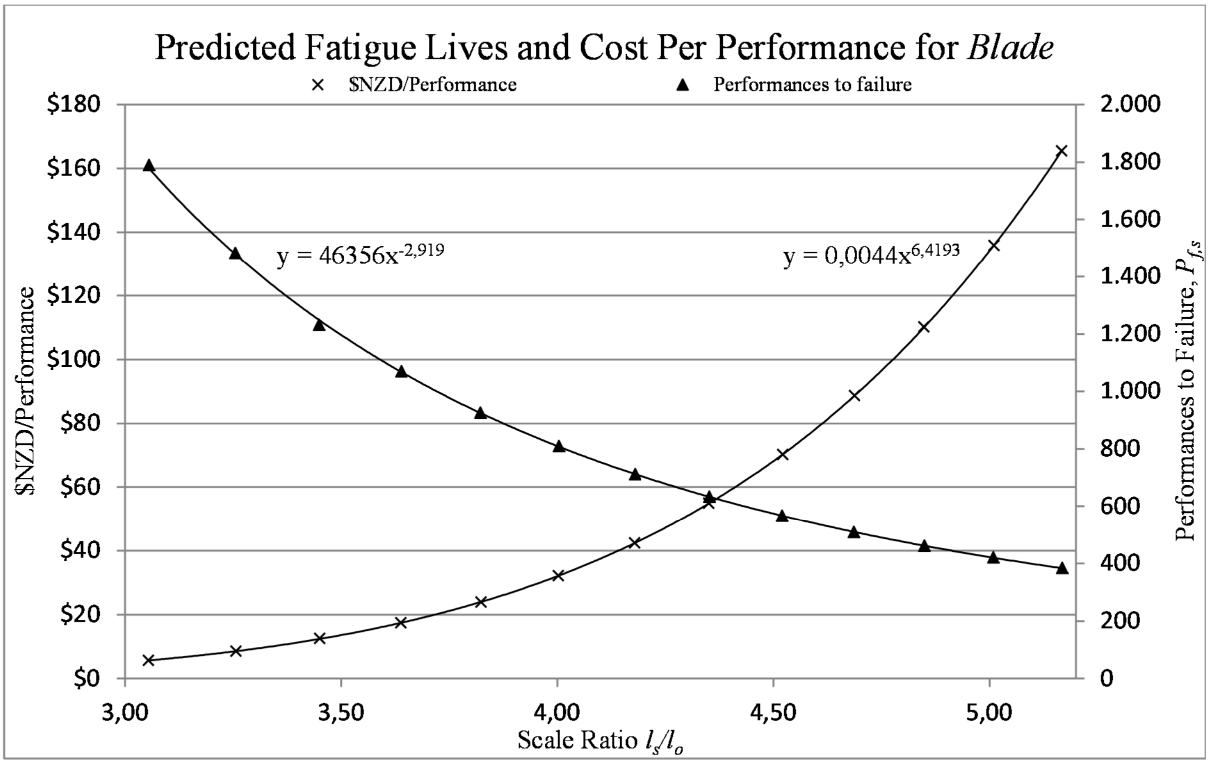

Figure 9. Trends for the predicted fatigue lives and cost per performance for increasing size of the scaled sculpture (based on price of titanium in China)

5. Discussion

5.1 Statistical results

The sample coefficient of variation is a measure of the uncertainty in the values of $P_{f,s}$ [Dowling 2007]. The SCOV remains around a value of 3.3 % for all free lengths of the blade material. This suggests that the consistency of the sculpture performances in the sample data for each blade free length is high i.e. the loading history experienced by the blade material during a performance is sufficiently repeatable. This result gives additional confidence to the one-sided tolerance limit lives presented in Table 1.

The one-sided tolerance limit lives account for the small sample size for fatigue life calculations. Each life predicted is three standard deviations below the mean life predicted to give a predicted fatigue life with 95 % confidence of 99 % reliability for the blade material.

5.2 Maximum scale

Several factors must be taken into account to determine a maximum possible scale for the *Blade* sculpture. A combination of material availability and a target price per performance will determine the practicality of building the maximum economic scale of *Blade*. The largest length of titanium material available at the correct thickness is 10024 mm. This would correspond to a free length for *Blade* of 8424 mm. At this size of blade material the cost per performance to the Len Lye Foundation in $165NZD. Compare this to the target cost per performance for Gooch's *Blade* of less than $500NZD per performance (in 1996) [Gooch 2001]. This would suggest that building *Blade* using the largest piece of titanium currently available is certainly practical in terms of the fatigue life of the blade material.

The maximum theoretical economic scale of *Blade* can be determined if manufacturing limitations are ignored. The equation for the cost per performance, C_P, in Figure 9 closely follows a power relationship of the form,

$$C_P = 0.0044\left(\frac{l_s}{l_o}\right)^{6.4193} \quad (8)$$

Using Lye's target of a 9m blade free length, equation 8 gives a cost per performance of approximately $255NZD. Therefore, based on the results of this fatigue life study, it is apparent that Lye's grand vision of a 9 m *Blade* is a theoretical possibility.

5.3 Increasing the fatigue life of the *Blade* material

A large number of post manufacture heat treatments exist for the titanium 6Al-4V alloy. Generally, the material is supplied in the mill annealed form where the optimum combination of strength and fatigue properties is obtained. Further heat treatments result in a trade-off of one of these properties in the material [Boyer et al. 1994]. Solution treating and aging the alloy will increase the endurance limit of the alloy along with its ultimate tensile strength at the expense of damage tolerance properties i.e. the alloy becomes brittle through the age hardening process. A re-crystallization anneal will improve damage tolerance properties at the expense of material strength. The re-crystallization anneal has surpassed beta annealing in industry for fracture critical airframe components which would suggest this heat treatment would be an ideal option for the blade material, should an increase in fatigue resistance properties be desired.

However, it would be prudent to investigate the effects of wind loading and any other unforeseen circumstances using a mill annealed blade for a new scaled sculpture before deciding on the usefulness of a further heat treatment. The loading measured in the test rig scales to significantly less than the yield strength of a mill annealed material but if any unforeseen loading was experienced by the blade material in a larger sculpture, the re-crystallization annealed item would have a lower material strength to sustain the loading. A mill annealed blade would have comparable properties to that of the material used in the S-N curve data source for this study and is, therefore, a sufficient heat treatment for the scaled sculpture using the largest piece of material available.

6. Conclusions

This paper has presented a study into the fatigue life of the blade material in scaled up versions of the Len Lye kinetic sculpture, *Blade*. A test rig was built at the scale of the original *Blade* prototype and the loading history of a typical performance was measured at the critical stress location in the blade material using strain gages. Fatigue lives were predicted for various scale ratios of the sculpture and it was found that:

1. The maximum economic scale ratio of *Blade* is 5.17:1, the scale which corresponds to a blade free length of 8.424 m. The Len Lye Foundation can be 95% confident that the sculpture will have a 99 % reliability of lasting 384 performances. This corresponds to a cost per performance of $165NZD.

2. The maximum theoretical economic scale ratio of *Blade* is 5.52:1, the scale which corresponds to a blade free length of 9m. This is the artistic scale of *Blade* which Len Lye intended. The cost per performance is predicted to be $255NZD.
3. The mill annealed heat treatment will be sufficient for the sculpture built to the maximum economic scale ratio.
4. The procedure used to determine the life of the blade material in this study can be extended to any flexible structure that exists in a public space undergoing variable amplitude stress cycles. Lives calculated using the above procedure are expected to be an accurate prediction using current fatigue theory.

Further work could consist of determining which part of a performance is most damaging to the blade material and potentially adjusting the mechanism of the sculpture or timing of certain parts of the performance to increase the life of the sculpture while maintaining artistic intentions.

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